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Optimal Risk Sharing through Renegotiation of Simple Contracts*

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We frequently observe that contracts do not include all of the contingencies that would seem to be necessary for optimal risk sharing between the parties to the contract. One reason may be that the possibility of renegotiation makes the contract more contingent than it appears. A simple contracting problem is used to show how even a simple contract may achieve optimal risk sharing if new information arrives slowly relatively to the speed of renegotiation. *Journal of Economic Literature* Classification Numbers: C72,D60,D84.

I. INTRODUCTION

It is a commonplace that the contracts we observe in practice are simpler than the theory of optimal contracts would predict. In particular, contracts do not appear to exploit all the opportunities for risk sharing between the parties to the contract. There are undoubtedly many reasons for this. One possibility, which does not seem to have been explored in the literature, is that simple contracts may be sufficient for optimal risk sharing.

Suppose that two agents want to write an optimal contract for the production and delivery of some good. The cost of producing the good is uncertain and will not be known until after the contract is signed. Clearly,

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there is some advantage to sharing this risk. A contract with a fixed delivery price forces the supplier to bear the risk of fluctuations in cost. For this reason, we do not expect a fixed-price contract to be optimal. However, if the contract can be renegotiated as new information about production costs becomes available, there are circumstances in which a fixed-price contract is optimal. More precisely, if the amount of information that arrives at each stage of the renegotiation game is not too great, a simple contract may be sufficient to share the risks optimally. By piecing together these renegotiations in the right way, it may be possible to attain the first best.

This idea is analogous to the well-known result in financial economics that continuous trading of a small number of securities will make markets effectively complete.[1] There are two main differences, however. First, renegotiation involves strategic interaction between two players, whereas traders in competitive markets are assumed to behave nonstrategically. Second, the contracts that are used in the renegotiation game are nonlinear. The nonlinearity of these contracts may seriously restrict their ability to support optimal risk sharing, in contrast to the linear structure of competitive markets. These issues are discussed further in Section VI.

It is important to understand the sense in which simple contracts are claimed to be optimal in this paper. The possibility of renegotiation often guarantees that the outcome of a contract is efficient ex post, after all uncertainty has been resolved. This is trivially true in the present setting. But that is not what is being claimed. What is claimed is much stronger, namely, that renegotiation of simple contracts will achieve efficient risk sharing ex ante, before the true state of nature is known.

In many settings, the possibility of renegotiation is a constraint on the ability of the contracting parties to achieve an ex ante efficient outcome.[2] In the presence of moral hazard, for example, renegotiation restricts the contracting parties' ability to structure incentives optimally. Ex ante, they would like to commit themselves to punish any deviation from optimal behavior. Ex post, when such a deviation has occurred, it may be inefficient to carry out the punishment. If it is possible to renegotiate an inefficient outcome, rational agents will do so. But then, anticipating the possibility of renegotiation, the agents will not find the original contract credible, so it cannot provide the right incentives.

In contrast, the present paper deals with a pure risk-sharing problem. The objective is to achieve the first-best allocation of risk when there is complete information and no moral hazard. In this setting, the role of

[1] See Duffie (1988) for a comprehensive treatment of this literature.

[2] See Dewatripont (1989) and Hart and Tirole (1988) for examples of models in which the possibility of renegotiation may lower the welfare of the contracting parties.

renegotiation is to increase the variation of income across states of nature and this in turn allows greater risk-sharing. So renegotiation has only beneficial effects on the outcome of the contracting process.

The fact that renegotiation can sometimes improve matters has been recognized by several writers. Huberman and Kahn (1988) point out that contracts often contain clauses which are never intended to be enforced. The function of these clauses is to provide a threat point, allowing one of the parties to enforce good behavior by the other. In an unpublished paper which is closely related to the present paper, Huberman and Kahn (1986) present a definition of simple contracts and present a number of examples to show, in a strategic setting with no uncertainty, that simple contracts with renegotiation can sometimes do as well as more complex contracts with no renegotiation.

Hart and Moore (1988) argue that renegotiation provides a mechanism for making a contract effectively contingent on information that cannot be verified by a third party, such as a court of law. In some circumstances, these mechanisms can even achieve the first best, though this is not true in general. Aghion *et al.* (1991) have extended the Hart–Moore analysis to show that quite simple mechanisms can achieve the first best in some circumstances, but they are not simple contracts in the sense of the present paper.

Hermalin and Katz (1990) provide another example of how renegotiation can improve welfare when there is asymmetric information. Chung (1990) studies renegotiation of simple contracts when parties are risk neutral and make specific investments.

A related literature has studied the problem of lack of commitment (Malcomson and Spinnewyn, 1988; Rey and Salanie, 1990). When agents cannot make long-term contracts, they may be forced to use a sequence of short-term contracts. Under certain conditions, a sequence of short-term contracts can do as well as a long-term contract. The essential requirement is that each short-term contract must specify the correct status quo point for the negotiation of the subsequent contract. In the setting studied below, there is no problem of commitment. The focus is instead on the simplicity of the contract relative to the information available.

II. THE MODEL

The general idea will be illustrated by a simple contracting problem. It should be clear from the discussion of this particular problem how the analysis can be generalized. Some extensions are considered at the end of the paper.

Suppose there are two agents, a *purchaser* and a *supplier,* who want to write a contract for the delivery of a good. For concreteness, we can think of the purchaser as the manufacturer of a durable consumer good and the supplier as the manufacturer of a component. At the time the contract is written, the two agents may be well informed about the technical specification of the component, but quite uncertain about the cost of producing the component or the value of the final product. There would seem to be some gains from sharing these risks.

To be more precise, suppose there is a finite number of states of nature, indexed by $i = 1, \ldots, I$. The probability that state i occurs is $a_i > 0$.

One unit of the good is worth $v_i > 0$ units of money to the purchaser in state i. There is no alternative use for the good, so only the purchaser values it.

The supplier uses l units of labor to produce $f(l)$ units of the good in any state. The production function f has the usual (neoclassical) properties:

Assumption 1. $f\colon \mathbb{R}_+ \to \mathbb{R}_+$ is C^2 with $f'(l) > 0$ and $f''(l) < 0$ for any $l > 0$. Also, $f'(l) \to \infty$ as $l \to 0$ and $f'(l) \to 0$ as $l \to \infty$.

The money wage in state i is ω_i. If l units of labor are used to produce the good, the total surplus generated is $v_i f(l) - \omega_i l$.

The Production Decision

Once the state of nature is known, it is in the interests of both agents to maximize the total surplus. Let S_i^* denote the maximized surplus in state i. Without loss of generality, the states of nature can be ordered so that

$$S_{i+1}^* > S_i^* \qquad \text{for } i = 1, \ldots, I - 1,$$

that is, higher states are better states.

Suppose that state i occurs and the agents have agreed on a delivery price $p \geq 0$. Suppose further that the choice of production level is left to the supplier. The supplier chooses $l \geq 0$ to maximize $pf(l) - \omega_i l$. From Assumption 1, it is clear that there is a unique optimal value of l, call it $l_i(p)$, for each state i and delivery price p. Conversely, for any production decision l there is a unique price p such that $l = l_i(p)$, that is, a unique price supporting the choice of l in state i. Thus, instead of specifying a production level for each state of nature, a contract could just as well specify a delivery price and leave the choice of production level to the supplier.

If the delivery price is p, the supplier's profit in state i is denoted by $\pi_i(p)$, where

$$\pi_i(p) \equiv pf(l_i(p)) - l_i(p);$$

the purchaser's income is denoted by $w_i(p)$, where

$$w_i(p) \equiv (v_i - p)f(l_i(p)).$$

The functions $\pi_i(\cdot)$ and $w_i(\cdot)$ are both continuous. It is assumed that higher states are more profitable for the supplier:

Assumption 2. $\omega_{i+1} < \omega_i$, for $i = 1, \ldots, I - 1$.

Then, from the definition of the profit function $\pi_i(p)$, it is easy to show that

$$\pi_{i+1}(p) - \pi_i(p) \geq (\omega_i - \omega_{i+1})l_i(p).$$

From the first-order conditions, $l_i(p) \to \infty$ as $p \to \infty$, so Assumption 2 implies that

$$\pi_{i+1}(p) - \pi_i(p) \to \infty \qquad \text{as } p \to \infty.$$

This property will prove useful later on.

Risk Preferences

Each agents' risk preferences are represented by a von Neumann–Morgenstern utility function. The purchaser's utility is $U(w)$ and the supplier's utility is $V(w)$, where in each case w denotes the agent's final wealth measured in terms of money.

Assumption 3. U, V: $\mathbb{R} \to \mathbb{R}$ are C^2 functions with $U'(w)$, $V'(w) > 0$, $U''(w) \leq 0$ and $V''(w) < 0$ for any w. Also, $\liminf_{w} V'(w) > 0$.

The last part of Assumption 3 ensures that the supplier's utility function is unbounded above and below. This is needed to ensure that a certain spanning condition is satisfied.

Complete Contracts

Before the true state is known, the two agents have an incentive to share risks. They can do this by writing a contract specifying how much will be produced in each state and how the surplus will be divided. It is assumed that contracts are binding, in the sense that the courts can observe whether the good has been delivered and will enforce the terms of the contract at zero cost.

A complete contract specifies a production decision and a sharing rule for each state. From the preceding discussion, there is no loss of generality in representing a complete contract by a price and a lump sum transfer to the supplier, in each state. Then a *complete contract* is defined by an array $\{(p_i, m_i)\}$, where p_i is the delivery price and m_i the lump sum transfer in state i. In the usual way, m_i is negative if there is a transfer from the supplier to the purchaser.

An *efficient* contract $\{(p_i^*, m_i^*)]$ solves the problem

$$\operatorname*{Max}_{\{(p_i, m_i)\}} \sum_{i=1}^{I} a_i U(w_i(p_i) - m_i) \qquad \text{s.t.} \sum_{i=1}^{I} a_i V(\pi_i(p_i) + m_i) \geq \bar{v},$$

for some constant $\bar{v}$. From the maintained assumptions, there is a unique efficient contract $\{(p_i^*, m_i^*)\}$ for each value of $\bar{v}$. A necessary condition for the contract to be efficient is that surplus be maximized in each state. From the maintained assumptions, it is clear that there is a unique level of production in each state that maximizes surplus. To achieve this level of production, the contract must have $p_i^* = v_i$ in state i. Note also that $\pi_i(p_i^*) + m_i^*$ is increasing in i. This follows from the maintained assumptions and the first-order conditions for efficient risk sharing.

Simple Contracts

Now suppose the two agents write a simple contract for delivery of the good. Rather than offer a general definition of "simplicity," I propose to look at a special case that clearly satisfies our intuitive notion of what constitutes a simple contract. For the purposes of this exercise, I assume a simple contract specifies a fixed delivery price p. The supplier chooses how much of the good to supply in each state and the purchaser is committed to accepting (and paying for) this quantity of the good. In addition, the agents can make a side payment at the time the contract is signed.

The assumption that the supplier chooses the quantity of the good may seem restrictive here, but it will be clear from what follows that there is no loss of generality in this. If we can prove the theorem under this assumption, it will hold a fortiori if the contract specifies the level of output as well. A *simple contract* is defined to be an ordered pair (p, m), where p is the delivery price and m is the transfer to the supplier at the time the contract is signed. A negative value of m indicates a transfer to the purchaser.

The purchaser's expected utility from a simple contract (p, m) is

$$\sum_{i=1}^{I} a_i U(w_i(p) - m)$$

and the supplier's expected utility is

$$\sum_{i=1}^{I} a_i V(\pi_i(p) + m).$$

It should be clear that a simple contract cannot, in general, implement the first best. For example, if the purchaser is risk neutral the supplier should bear none of the risk. This is impossible under the simple contract described above, since the fixed delivery price implies the supplier bears some of the risk from variations in labor costs. To achieve the first best, two additional ingredients are needed: the possibility of renegotiation and a special information structure.

III. THE RENEGOTIATION GAME

The Information Structure

To allow for the possibility of renegotiation, I assume there is an interval between the time the contract is signed and the time the good is produced. This interval is divided into a finite number of *dates,* indexed by $t = 0, 1, \ldots, T$. At each date, the agents receive information about the true state of nature. The information available to the two agents at any date t is called their information set and is represented by a node in a partially ordered graph. Let N_t denote the information sets (nodes) at date t and let $N = \bigcup N_t$ denote the set of all information sets. There is a unique initial information set, n_0, at the first date. The information at the first date is summarized by the probability distribution $\{a_i\}$. As time passes, information about production costs and the value of the final product accumulates steadily. At the last date, the true state of nature is revealed. At each information set, the agents update their prior probability distribution to take account of their new information about the true state of nature. Let a_{in} denote the probability of state i conditional on reaching node n. Formally, the preceding assumptions can be summarized:

$$N_0 = \{n_0\} \text{ and } a_{in_0} = a_i > 0 \qquad \text{for every } i = 1, \ldots, I.$$

$$N_T = \{0, \ldots, I\} \text{ and } a_{in} = \begin{cases} 1 & \text{if } i = n \\ 0 & \text{if } i \neq n \end{cases} \qquad \text{for every } i, n \in N_T.$$

The preceding assumptions are fairly innocuous. The next assumption is crucial: I assume that the graph has a *binomial* structure. Suppose that $n \in N_t$ and $n' \in N_{t+1}$, for some $t = 0, \ldots, T - 1$. Call n' a *successor* of n and n a *predecessor* of n' if the probability of n' being observed at date $t + 1$ is positive, conditional on n being observed at date t.

Assumption 4. Any node $n \in N \backslash N_T$ has precisely two successors, labeled $n+$ and $n-$. Any node $n \in N \backslash N_0$ has a unique predecessor.

Assumption 4 is intended to capture the idea that information arrives slowly (or at least not too quickly) relative to the renegotiation process. The probabilities of $n+$ and $n-$ conditional on n are denoted by π_{n+} and π_{n-}, respectively.

Finally, I make another simplifying assumption. Let $\{A_{in}\}$ denote the cumulative probability distribution of the true state conditional on the information set $n \in N$. That is,

$$A_{in} = \sum_{j \leq i} a_{jn} .$$

Then for any information set n, it is assumed that $\{A_{in+}\}$ is better than $\{A_{in-}\}$ in the sense of first-order stochastic dominance.

Assumption 5. For any information set $n \in N \backslash N_T$ and any state $i = 1, \ldots, I$, $A_{in+} \leq A_{in-}$.

The Renegotiation Game

At date 0, some kind of bargaining process takes place. We do not need to model this explicitly. We simply assume that it results in a contractual price p^0 and a lump sum transfer m^0. In defining the renegotiation game, p^0 and m^0 are treated as exogenous parameters.

Assuming the initial parameters (p^0, m^0) have been chosen optimally, there is no need for renegotiation at the first date. At subsequent dates, renegotiation will typically occur. The agents are allowed to replace the existing contract price with a new price if they both agree. They can also make side payments. The renegotiation procedure is modeled in the simplest possible way. Suppose that some date t has arrived and both agents have observed an information set n. The *status quo* is an ordered pair (p, m) representing the price prevailing at the beginning of the period and the net transfer received by the supplier to date. The purchaser makes a *proposal* to the supplier in the form of a new contract price p' and a new net transfer m'. The supplier can accept or reject the proposal. If accepted, the new contract price p' replaces the old price and a side payment $(m' - m)$ is made to the supplier. If the proposal is rejected, the existing contract price remains in force and there is no side payment.

At the end of the last date, after the true state has been revealed and renegotiation has taken place for the last time, the supplier makes the production decision, delivers the good, and receives the contracted price. Note that there is scope for renegotiation at the last date. The contract price p may not induce an efficient production decision. In that case, it is efficient to change the price and make a side payment to compensate the injured party.

Equilibrium

Each player is assumed to have complete information and perfect recall. A strategy for a player specifies a feasible action at each node of the game controlled by that player. The purchaser's strategy specifies a proposal at each date $t > 0$. The supplier's strategy specifies her decision to accept or reject the purchaser's proposal at each date and a production decision at the final date. For each pair of strategies, there is a unique outcome in each state of nature at date T and a unique payoff for each player. The game defined in this way is denoted by $\Gamma(p^0, m^0)$.

We analyze this game using the concept of *Subgame Perfect Equilibrium* (SPE). A SPE is a pair of strategies such that, for each node of the game, the strategy of the agent who controls that node is a best response to the other player's strategy, in the subgame defined by that node.

IV. OPTIMAL RISK SHARING

Suppose that $\{(p_i^*, m_i^*)\}$ is an efficient contract and suppose that (σ, τ) is a SPE of the renegotiation game $\Gamma(p^0, m^0)$. For any pair of strategies and any initial state, there is a unique outcome in the renegotiation game. Let (p_i, m_i) denote the outcome in state i under the strategies (σ, τ). The equilibrium *implements* the efficient contract if and only if $(p_i, m_i) = (p_i^*, m_i^*)$ for $i = 1, \ldots, I$. Obviously, if the SPE implements the efficient outcome, the efficient production decision will be made in each state and both purchaser and supplier will get their first-best payoffs. The converse is also true: If (σ, τ) yields the first-best payoffs to each player, then the outcome must be (p_i^*, m_i^*) in each state i.

Under the maintained assumptions, it is possible to implement efficient risk sharing as a SPE of the renegotiation game.

THEOREM. *Let $\{(p_i^*, m_i^*)\}$ be a complete, efficient contract. Then there exists a simple contract (p^0, m^0) such that the renegotiation game $\Gamma(p^0, m^0)$ has a SPE and any SPE of $\Gamma(p^0, m^0)$ implements $\{(p_i^*, m_i^*)\}$.*

The proof is contained in the Appendix. To illustrate the argument of the proof, some examples are considered in the next section.

V. EXAMPLES

A Two-State Example

To start with the simplest possible case, consider an example in which there are only two states of nature, $i = 1, 2$, and two dates, $t = 0, 1$. The

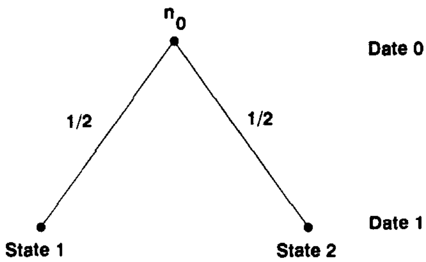

FIG. 1. There are two dates, $t = 0, 1$, and two states, $i = 1, 2$. At the first date, each state appears equally likely. At the second, the true state is revealed.

states are equally likely, so $a_i = \frac{1}{2}$ for $i = 1, 2$. Also, assume that $v_i = 1$ for $i = 1, 2$.

The information structure is shown in Fig. 1. At the first date, the agents only know the probability distribution $\{a_i\}$. At the second date, they know the true state.

It is easy to see that, without renegotiation, a simple contract cannot implement the first best except by accident. Suppose that the parties agree to a fixed delivery price p and a net transfer m at date 0. A necessary condition for the first best to be achieved is that $p = 1$, since otherwise the supplier will not choose the optimal production level. With this delivery price, the supplier's final wealth is $S_i^* + m$ in state i. In other words, the supplier bears all the risk of fluctuations in costs. In general, this will not be optimal. For example, if the purchaser is risk neutral and the supplier risk averse, the purchaser should bear all the risk and the supplier should bear none.

Now suppose that the simple contract can be renegotiated at the second date. Then the final outcome will have two important properties. *First,* whatever contract (p, m) is agreed at date 0, the outcome will be efficient at date 1. *Second,* the supplier will be no better off, in each state, than if no renegotiation had occurred. To see this, suppose that a contract (p, m) has been written at date 0 and some state i has been revealed at date 1. Clearly, if $p = 1$ there is no scope for renegotiation, because no proposal the purchaser can make will improve his own payoff without making the supplier worse off. So we may as well assume that $p \neq 1$. Then the purchaser can make a proposal (p', m') that makes both parties better off. The unique equilibrium proposal in this subgame is defined by the conditions

$$p' = 1$$

and

$$\pi_i(p) + m = \pi_i(p') + m'.$$

The new price is chosen to maximize surplus and the transfer is chosen so that the supplier's utility is exactly the same as before. All the gains from the renegotiation go to the purchaser. This proposal is clearly optimal for the purchaser, if the supplier is willing to accept it. It is also weakly optimal for the supplier to accept the proposal, since the alternative of rejecting it is no better for her. Furthermore, the supplier must accept the proposal in any SPE, because it is the limit of a sequence of proposals each of which makes the supplier strictly better off. Since the supplier must accept each of these proposals in a SPE, the purchaser can achieve the payoff offered by (p', m'). Since (p', m') is the unique, acceptable proposal that yields this payoff to the purchaser, it must be accepted in equilibrium.

The possibility of renegotiation ensures an efficient outcome in each state, regardless of the initial contract. The role of the initial contract (p, m) is simply to determine the supplier's payoff. It acts like the *status quo* point in the bargaining game that chooses the new contract at date 1. Suppose that by an appropriate choice of (p, m), we could make the supplier's payoff in each state equal to her first-best payoff. Since the choice of any contract leads to a unique, efficient outcome in each state, the unique SPE of the renegotiation subgame must implement the first best. The fact that the supplier gets her first-best payoff is necessary and sufficient for implementation of the first best.

It remains to be shown that an appropriate choice of (p, m) will give the supplier the right payoffs in each state. Let $\{(p_i, m_i)\}$ be the first-best contract and let the supplier's first-best wealth be $y_i = \pi_i(p_i) + m_i$, in each state $i = 1, 2$. To implement the first best, it is necessary and sufficient to choose an initial contract with the property that

$$\pi_i(p) + m = y_i,$$

for $i = 1, 2$. Does such a contract exist? Under the maintained assumptions the answer is yes. From the conditions for first-best risk sharing, we know that $y_1 < y_2$. Also, $\pi_2(p) - \pi_1(p)$ grows unboundedly large as p approaches ∞. For any price p, choose the initial lump-sum transfer so that the contract gives the right payoff in state 2, that is, put

$$m(p) \equiv y_2 - \pi_2(p).$$

The contract we are looking for must also satisfy the condition that

$$m(p) = y_1 - \pi_1(p).$$

By varying p, we can satisfy this condition. For $p = 0$,

$$\pi_1(p) + m(p) = m(p) = y_2 > y_1.$$

On the other hand, as $p \to \infty$, $\pi_2(p) - \pi_1(p) \to \infty > y_2 - y_1$ so eventually,

$$\pi_1(p) + m(p) < \pi_2(p) + m(p) - (y_2 - y_1) = y_1.$$

For some intermediate value of p the required condition must be satisfied. For this choice of p, there is a unique SPE of $\Gamma(p, m(p))$ and it implements the first best.

A Four-State Example

The preceding example captures most aspects of the proof of the general case and, in particular, it illustrates the crucial role of the binomial information structure. Because there are only two states, it is possible to choose the two parameters of the initial contract so that the supplier has the right reservation utility in the renegotiation game in each state. Roughly speaking, we choose m so that the supplier has the right average utility and then choose p so that she has the right difference in utility between the two states. What the example does not illustrate, because there is only renegotiation at one date, is the way in which successive rounds of renegotiation allow a simple contract to achieve optimal risk sharing when there are many states of nature. We can get some idea of how this works by considering a slightly more complex example. Suppose there are four states, $i = 1, 2, 3, 4$, and three dates, $t = 0, 1, 2$. As before, the states are assumed to be equally likely, so $a_i = \frac{1}{4}$ for $i = 1, \ldots, 4$. The information structure is illustrated in Fig. 2 below. At the first date, the agents know only the initial probabilities $\{a_i\}$. At the second date, they learn either that the true state belongs to $\{1, 2\}$ or that it belongs to $\{3, 4\}$. At the last date, they learn the true state.

The nodes labeled A, B, and C correspond to the information sets $\{1, 2, 3, 4\}$, $\{1, 2\}$, and $\{3, 4\}$, respectively. Conditional on the information available at node A, nodes B and C are equally likely. Conditional on the information available at node B, states 1 and 2 are equally likely. Likewise, at node C, states 3 and 4 are equally likely. In terms of the notation introduced earlier,

$$A+ = C,\ A- = B,\ B+ = 2,\ B- = 1,\ C+ = 4,\ C- = 3$$

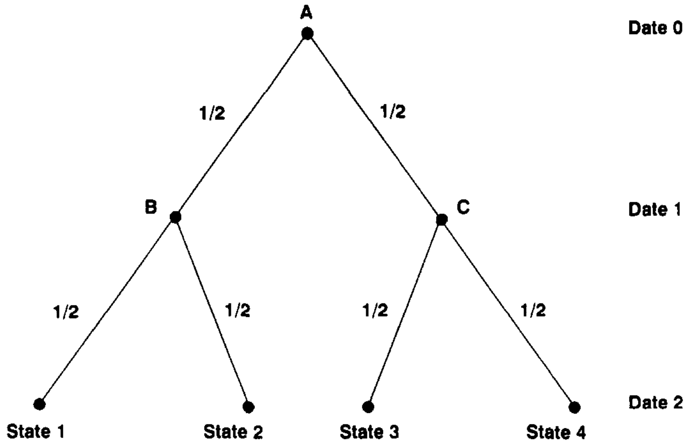

FIG. 2. There are three dates, $t = 0, 1, 2$, and four states, $i = 1, 2, 3, 4$. At the first date, nodes B and C are equally likely. At the second date, if node B (resp. C) is reached, states 1 and 2 (resp. 3 and 4) are equally likely. At the last date, the true state is revealed.

and

$$\pi_{A+} = \pi_{A-} = \pi_{B+} = \pi_{B-} = \pi_{C+} = \pi_{C-} = \frac{1}{2}.$$

Let y_i denote the wealth of the supplier in state i under the first-best contract. Then the construction we used earlier can be used iteratively to show that the first best can be achieved by successive renegotiations of the contract.

Suppose node B has been reached. We have shown that there is a simple contract (p^B, m^B) with the property that, if this contract is adopted at node B, the unique equilibrium of the subgame starting at B will implement the first-best allocation in states 1 and 2. Similarly, if C is reached, there is a simple contract (p^C, m^C) with the property that, if this contract is adopted at node C, the unique equilibrium of the subgame starting at C will implement the first-best allocation. Thus, in order to achieve the first best, it is sufficient to show that there is a simple contract (p^A, m^A) which, if it were adopted at node A, would lead to the adoption of (p^B, m^B) and (p^C, m^C) at nodes B and C, respectively.

To show that this is possible, first compute the first-best payoffs for the supplier, conditional on nodes B and C being reached, respectively:

$$\bar{v}_B = \frac{1}{2}\{v(y_1) + v(y_2)\} \quad \text{and} \quad \bar{v}_C = \frac{1}{2}\{v(y_3) + v(y_4)\}.$$

The question now is whether there exists a contract (p^A, m^A) which would be worth $\bar{v}_B$ (resp. $\bar{v}_C$) to the supplier if node B (resp. node C) were reached at date 1 and the initial contract (p^A, m^A) were not renegotiated. To answer this question, we need to calculate the value of (p^A, m^A) to the supplier at each of the nodes. The calculation is made easier by the fact that, even if renegotiation occurs at the last date, it will not make the supplier better off. Thus, in calculating the supplier's reservation utility there is no loss of generality in assuming the contract is not renegotiated at any date. In that case, the reservation utility at node B, denoted $v_B(p^A, m^A)$, is defined by

$$v_B(p^A, m^A) = \frac{1}{2}\{V(\pi_1(p^A) + m^A) + V(\pi_2(p^A) + m^A)\}.$$

Similarly, the reservation utility at node C, denoted $v_C(p^A, m^A)$, is defined by

$$v_C(p^A, m^A) = \frac{1}{2}\{V(\pi_3(p^A) + m^A) + V(\pi_4(p^A) + m^A)\}.$$

From the assumption that higher states are better states it follows that $\bar{v}_B$ is greater than $\bar{v}_C$. From the assumption that profits are higher in higher states it follows that $v_B(p^A, m^A)$ is greater than $v_C(p^A, m^A)$. Then we can use the argument from the two-state case to show that there exists a contract (p^A, m^A) satisfying

$$v_B(p^A, m^A) = \bar{v}_B$$

and

$$v_C(p^A, m^A) = \bar{v}_C.$$

Essentially, we choose m^A so that the supplier gets the right expected utility on average and then choose p^A so that the difference between the expected utilities at the two nodes is right.

Once we have found the right initial contract (p^A, m^A), we can show that any SPE of $\Gamma((p^A, m^A))$ will implement the first best. Whichever node is reached at date 1, the purchaser will propose (p^B, m^B) or (p^C, m^C) as appropriate and the supplier will accept. It is optimal for the supplier to accept because each of these contracts leads to a unique outcome at the last date in which she receives the same payoff as if there had been no renegotiation. The proposal is optimal for the purchaser, since it gives him the first-best payoff and this is the best he can achieve without making the supplier worse off. (The supplier will never agree to anything that makes her worse off.) Furthermore, this is the only possible outcome given the initial contract (p^A, m^A). There are sequences of proposals converging to (p^B, m^B) and (p^C, m^C), respectively, which have the property that each of the proposals makes the supplier strictly better off than her reservation expected utility and which must therefore be accepted in a SPE. This proves that any SPE starting from the initial contract (p^A, m^A) must implement the first best.

This construction of a SPE implementing the first-best makes use of two basic facts. The first is that, whatever state occurs and whatever contract is in force, renegotiation at the last date will ensure that the outcome is efficient ex post. The second is that, at each date, the purchaser's proposal will give the supplier her reservation level, that is, the payoff she would get from the continuation game if she rejected the proposal. The problem of constructing a SPE to implement the first best reduces to the problem of finding a sequence of proposals such that, at each information set, the supplier's reservation expected utility is equal to her first-best payoff conditional on that information set. More precisely, the proposal (p, m) accepted at node n must have the property that, at each of the nodes $n+$ and $n-$, the reservation expected utility determined by (p, m) is equal to the supplier's expected utility under the efficient contract, conditional on those information sets. Then

(i) it will be optimal for the supplier to accept these proposals, since she is no worse off;

(ii) ex post efficiency will ensure that the purchaser also achieves his first-best payoff in each state;

(iii) it will be optimal for the purchaser to make these proposals at each information set since, by the definition of a complete, efficient contract, he cannot do better without forcing the supplier below her reservation expected utility at some point.

This sketch shows how the equilibrium *path* can be constructed. To describe the entire equilibrium, it is necessary to specify strategies off the equilibrium path. The construction is the same: the equilibrium of any subgame is uniquely defined by the payoffs associated with the contract in force at the initial node of the subgame. The details are found in the Appendix.

VI. DISCUSSION

The Spanning Condition

The essential conditions needed to achieve efficient risk-sharing are (i) the possibility of renegotiation and (ii) the assumption that information arrives slowly relative to the renegotiation process. As the examples presented in Section V reveal, the second of these conditions is captured in the binomial information structure. At each node of the information structure, the contract only has to "span" the two branches issuing from that node. The two parameters p and m give the contract enough flexibility to do that. Without this property, it would be impossible to implement the first best. Let us call the requirement for the contract to "span" the two branches the *spanning condition*.

Formally, let v_n denote the first-best payoff of the supplier conditional on the node n being reached:

$$v_n \equiv \sum_{i=1}^{n} a_{in} V(\pi_i(p_i) + m_i) \qquad \text{for } n \in N$$

Let $v_n(p, m)$ denote the equilibrium payoff conditional on node n being reached and on (p, m) being the contract in force at that time:

$$v_n(p, m) \equiv \sum_{i=1}^{n} a_{in} V(\pi_i(p) + m) \qquad \text{for } n \in N$$

Then the spanning condition is the requirement that, at each node n, there exists a contract (p^n, m^n) with the property that

$$v_{n+}(p^n, m^n) = \bar{v}_{n+}$$

and

$$v_{n-}(p^n, m^n) = \bar{v}_{n-}.$$

The assumptions maintained in the paper are sufficient to guarantee that the spanning condition always holds, but there are circumstances in which the spanning condition may fail. Consider the two-state example from Section V, but instead of having the purchaser make all the offers, assume that the supplier makes all the offers in the renegotiation game. Everything else is unchanged. Changing this one assumption is sufficient to cause the spanning condition to fail.

For concreteness, assume that $f(l) \equiv 2l^{1/2}$. Then it is easy to calculate that, for any price p,

$$l_i(p) = (\omega_i/p)^{-2}$$
$$w_i(p) = 2(1 - p)p/\omega_i$$

and

$$S_i^* = 2(\omega_i)^{-1} - \omega_i(\omega_i)^{-2} = (\omega_i)^{-1},$$

for $i = 1, 2$. Suppose that the purchaser is risk neutral and the supplier is risk averse. In that case, optimal risk sharing requires that the purchaser bear all the risk. Let w_i denote the first-best wealth the purchaser would receive under a complete, efficient contract. Since the purchaser bears all the risk,

$$w_2 - w_1 = S_2^* - S_1^*.$$

Because the supplier makes all the offers, the purchaser is held down to his reservation utility in every renegotiation session. The spanning condition now requires the initial contract (p, m) to give the purchaser his first-best payoff in each state:

$$w_i = w_i(p) - m.$$

Substituting from the formula for $w_i(p)$, we find that

$$w_2 - w_1 = 2(1 - p)p(1/\omega_2 - 1/\omega_1).$$

But

$$S_2^* - S_1^* = 1/\omega_2 - 1/\omega_1,$$

so the spanning condition can be satisfied if and only if

$$2(1 - p)p = 1.$$

Since $(1 - p)p \leq \frac{1}{4}$ for any $0 \leq p \leq 1$, this condition can never be satisfied. As a result, the first best can never be implemented with this form of contract, when the offers are made by the supplier. The problem is that when the supplier chooses the level of production, fixing the delivery price does not provide sufficient variability in the purchaser's wealth to span the required income levels in the two states. Whatever initial contract is chosen, the purchaser will end up with the wrong reservation utility in at least one state. Renegotiation ensures that an efficient production level will be chosen in each state, but the risk sharing will be suboptimal.

Although the spanning condition cannot be satisfied by this particular contract when the supplier makes all the offers, another simple contract will do the job. We need to assume that $v_1 \neq v_2$, which is a mild regularity condition. Instead of specifying a fixed delivery price, a simple contract now specifies a fixed quantity of the good to be delivered. Then a contract is an ordered pair (l, m), where $f(l)$ is the quantity to be delivered and m is the transfer to the supplier. The final wealth of the purchaser in state i will be

$$v_i f(l) - m = w_i \qquad \text{for } i = 1, 2.$$

These equations can be solved for a unique value of (l, m) as long as $v_1 \neq v_2$.

The example teaches us two important lessons. First, the spanning condition is a nontrivial condition that may well fail to be satisfied for a particular contract and a particular game form. Second, even if one form of contract fails to satisfy the spanning condition, another form of contract is likely to satisfy it. The theory is robust in the sense that it is generally possible to find some form of simple contract and some renegotiation game that will implement the first best. The interesting question, then, is which contracts and which renegotiation games will lead to optimal risk sharing and what relation they bear to contracts used in practice.

Extensions

I have only explored the implementation of optimal risk-sharing in one special setting, but the argument could be applied in a variety of settings. The most obvious extensions include the following:

—allowing offers to be made by the purchaser;
—allowing alternating offers by the purchaser and supplier;
—allowing alternative contractual forms;
—allowing richer stochastic structures.

The first three extensions do not pose great difficulties. As long as the basic ingredients are present, i.e., the possibility of renegotiation and the special information structure, the implementation of the first best will follow. The crucial requirement is to show that the spanning condition is satisfied. This may not always be the case, but it will be satisfied for a large class of parameter values and forms of contract. If the spanning condition is satisfied, then at each node it is possible to choose contracts in such a way that the equilibrium of the continuation game at that node leads to the first best. Any contractual form that satisfies the spanning condition and any bargaining game that can be solved by backward induction will support the kind of result obtained in this paper.

Generalizing the information structure poses deeper problems. The binomial structure used in the paper is extremely special. One may not be especially concerned about this because a large class of stochastic processes (the diffusion processes) can be approximated by binomial processes. Arguing by analogy with the financial literature, it should be possible to show for any diffusion process that, if one can renegotiate sufficiently frequently, it will be possible to approximate the efficient outcome arbitrarily closely.

The extension in this direction clearly poses some technical challenges. Formally, there are two ways of dealing with diffusion processes. One is to start with a discrete number of renegotiation dates and then take limits as the time between renegotiations approached zero. The other would be to define a SPE in the limit, when renegotiation takes place continuously. The former approach seems more tractable, although it may be messy. The latter raises difficult conceptual as well as mathematical problems. For example, how would one deal with alternating offers?

APPENDIX

Preliminaries

The theorem is proved by induction. It is easier to prove a slightly stronger result, since a stronger induction hypothesis is needed for one part of the proof. But first, some preliminary definitions are required.

For any information set n and any *status quo* (p, m), let $\Gamma_n(p, m)$ denote the subgame that begins with the supplier's proposal at the node of the game defined by the information set n and the *status quo* (p, m).

For any information set n and *status quo* (p, m), let

$$V_n^*(p, m) = \sum_{i=1}^{I} a_{in} V(\pi_i(p) + m).$$

$V_n^*(p, m)$ is the payoff the supplier would receive if she started at the information set n with the *status quo* (p, m) and there were no renegotiation at n or any information set following n.

Define a complete contract $\{(p_i, m_i)\}$ to be *efficient conditionally on n* if it maximizes

$$\sum_{i=1}^{I} a_{in} U(w_i(p_i) - m_i)$$

subject to the constraint that

$$\sum_{i=1}^{I} a_{in} V(\pi_i(p_i) + m_i) \geq \bar{v}$$

for some constant $\bar{v}$. This is just the usual definition of an efficient contract, with conditional probabilities substituted for unconditional ones. Call a SPE of the subgame $\Gamma_n(p, m)$ *efficient* if it implements a contract that is efficient conditionally on n. (Here we ignore states that occur with zero probability.)

Now we can state the main lemma:

LEMMA. *For any status quo (p, m) and for any information set n, there exists an efficient SPE of the subgame $\Gamma_n(p, m)$ in which the supplier receives the payoff $\bar{v} = V_n^*(p, m)$. Furthermore, any SPE of $\Gamma_n(p, m)$ is efficient and gives the supplier the same payoff.*

Proof of the Lemma. The lemma is proved by backward induction. Suppose the last date $t = T$ has arrived, the information set $n = i$ has been reached, and the *status quo* is (p, m). If $p \neq v_i$, the supplier will not be willing to choose the optimal production decision so the agents' joint surplus will not be maximized. This means the purchaser can improve his payoff by making a proposal (p', m') that leads to an efficient production decision and makes the supplier no worse off.

Define a SPE of the subgame $\Gamma_i(p, m)$ as follows. Let $\bar{v} = V_i^*(p, m)$. The purchaser makes a proposal (p', m'), defined by putting

$$p' = v_i$$

and

$$m' = \pi_i(p) + m - \pi_i(p').$$

The supplier accepts any proposal (p'', m'') such that $\bar{v} \leq V_i^*(p'', m'')$. Whichever price obtains after the renegotiation phase, the supplier will choose an optimal production decision taking the price as given.

To see that this is a SPE, note first of all that whatever contract is in force, the supplier chooses a production decision that is optimal for her. Second, the supplier will get at least $\bar{v}$ if she accepts the proposal and will get exactly $\bar{v}$ if she rejects it. So the supplier's behavior is clearly optimal Finally, the purchaser's proposal leads to an efficient outcome and clearly maximizes his payoff, subject to the constraint that the supplier get at least $\bar{v}$. Thus all the conditions for a SPE are satisfied.

Furthermore, this is the only kind of equilibrium that is possible at date T. The supplier can always guarantee herself $\bar{v}$ by rejecting all proposals, so she will never get less than $\bar{v}$. On the other hand, the purchaser will never give her more. If he did, there would exist another proposal that would make him (the purchaser) better off and which the supplier could not refuse (because it gave her more than $\bar{v}$). The existence of such a proposal is inconsistent with equilibrium. Finally, by the same argument, the purchaser will always ensure that the outcome is efficient.

We have shown that the conditions of the Lemma are satisfied at date T. Now suppose that they are satisfied for dates t through T. At date $t-1$, consider any information set $n \in N_{t-1}$ and any *status quo* (p, m). Letting $\bar{v} = V_n^*(p, m)$, find a conditionally efficient contract $\{(p_i, m_i)\}$ that maximizes

$$\sum_{i=1}^{I} a_{in} U(w_i(p_i) - m_i)$$

subject to the constraint that

$$\sum_{i=1}^{I} a_{in} V(\pi_i(p_i) + m_i) \geq \bar{v}.$$

For this contract, let $\bar{v}_{n\cdot}$ denote the supplier's payoff conditional on the information set $n\cdot$, that is,

$$\bar{v}_{n\cdot} = \sum_{i=1}^{I} a_{in\cdot} V(\pi_i(p_i) + m_i),$$

for $n\cdot = n+, n-$. From the definition and Assumption 5, it follows that $\bar{v}_{n-} \leq \bar{v}_{n+}$. To see this, simply rewrite the expression for $\bar{v}_{n\cdot}$ as

$$\begin{aligned} \bar{v}_{n\cdot} = {} & V(\pi_1(p) + m) \\ & + \sum_{i=1}^{I-1} (V(\pi_{i+1}(p_{i+1}) + m_{i+1}) - V(\pi_i(p_i) + m_i))\,(1 - A_{in\cdot}). \end{aligned}$$

The desired result is immediate, because $\pi_{i+1}(p_{i+1}) + m_i \geq \pi_i(p_i) + m_i$ and $A_{in-} \leq A_{in+}$ for $i = 1, \ldots, I$.

To proceed with the proof, we need the following fact, which will be verified at the end.

CLAIM. *For any $n \in N \backslash N_T$, let $\bar{v}_{n+}$ and $\bar{v}_{n-}$ denote the efficient payoffs conditional on $n+$ and $n-$, respectively. Then there exists a proposal (p', m') satisfying*

$$\bar{v}_{n+} = V_{n+}^*(p', m') \qquad \text{and} \qquad \bar{v}_{n-} = V_{n-}^*(p', m').$$

Now we are in a position to define an efficient SPE of $\Gamma_n(p, m)$ that gives the supplier a payoff $\bar{v}$. The purchaser proposes (p', m') satisfying the conditions of the claim. The supplier accepts a proposal (p'', m'') if

$$\bar{v} \leq \pi_{n-} V^*_{n-}(p'', m'') + \pi_{n+} V^*_{n+}(p'', m'').$$

He rejects all other proposals. If a proposal (p'', m'') is accepted, then in the information sets $n-$ and $n+$, selected any SPE of the games $\Gamma_{n-}(p'', m'')$ and $\Gamma_{n+}(p'', m'')$, respectively. From the induction hypothesis, any SPE of the subgame $\Gamma_{n\cdot}(p'', m'')$ yields the supplier a payoff equal to $V^*_{n\cdot}(p'', m'')$. So her payoff if she accepts (p'', m'') will be $\pi_{n-} V^*_{n-}(p'', m'') + \pi_{n+} V^*_{n+}(p'', m'')$. If the proposal is rejected, then select any SPE of $\Gamma_{n-}(p, m)$ and $\Gamma_{n+}(p, m)$, respectively.

To see that we have defined an efficient SPE of $\Gamma_n(p, m)$, note first that whatever happens at date $t - 1$, we have by construction reached an efficient SPE of the subgame $\Gamma_{n\cdot}(p'', m'')$ at date t. Second, by construction, the supplier's strategy is optimal at date $t - 1$ since she will get at least $\bar{v}$ from the proposals she accepts and will only get $\bar{v}$ if she rejects. Finally, the purchaser's strategy must be optimal, for his proposal is accepted, yields an efficient SPE in each subgame at date t, and yields exactly $\bar{v}_{n\cdot}$ to the supplier in each information set $n\cdot$ at date t. The payoffs must therefore be efficient conditional on n and the purchaser will not be able to do better unless he can get the supplier to accept less than $\bar{v}$. But the supplier will never do this, since she is guaranteed $\bar{v}$ if she rejects the proposal.

To see that all SPE of $\Gamma_n(p, m)$ have the required properties, note that the supplier must get at least $\bar{v}$ in any SPE, since she can achieve this by rejecting all proposals. Second, the purchaser will never give her more than $\bar{v}$. This is because we can find a proposal (p'', m'') such that $\bar{v}_{n\cdot} < V^*_{n\cdot}(p'', m'') < \bar{v}_{n\cdot} + \varepsilon$ for any $\varepsilon > 0$ and $n\cdot = n-, n+$. The supplier must accept such a proposal and the resulting SPE, being efficient, must give the purchaser a payoff very close to his first-best payoff. Since $\varepsilon > 0$ is arbitrary, it is clear that in equilibrium the purchaser must get the efficient payoff, so the equilibrium is efficient and yields the supplier $\bar{v}$.

By induction, the properties of the Lemma must hold for all $t = 0, \ldots, T$.

Proof of the Theorem. To prove the theorem, let $\{(p_i^*, m_i^*)\}$ be an efficient contract and let $\bar{v}$ be the supplier's payoff. Since $\bar{v}$ is in the range of $V(\cdot)$, there exists a *status quo* (p^0, m^0) such that $\bar{v} = V^*_{n_0}(p, m)$. Then by the Lemma there exists an efficient SPE of $\Gamma(p^0, m^0)$ in which the supplier gets $\bar{v}$. Since the efficient payoffs are unique, the SPE implements $\{(p_i^*, m_i^*)\}$.

Furthermore, the Lemma implies that any SPE of $\Gamma(p^0, m^0)$ has the same payoffs and, since the payoffs are efficient, the outcome is uniquely determined; i.e., the outcome of any SPE will be (p_i, m_i) in each state i. Thus any SPE implements $\{(p_i^*, m_i^*)\}$.

This completes the proof of the Theorem.

Proof of the Claim. Let $\bar{v}_{n+}$ and $\bar{v}_{n-}$ be the efficient payoffs, conditional on $n+$ and $n-$, respectively, for some $n \in N \backslash N_T$ and $\bar{v}$. We need to show that, for some proposal (p, m),

$$\bar{v}_{n+} = V_{n+}^*(p, m) \qquad \text{and} \qquad \bar{v}_{n-} = V_{n-}^*(p, m).$$

If $\bar{v}_{n+} = \bar{v}_{n-}$ there is nothing to prove so we may assume without loss of generality that $\bar{v}_{n+} > \bar{v}_{n-}$.

Recall the following properties of the profit functions $\pi_i(p)$.

(i) $\pi_i(p)$ is continuous, $\forall i = 1, \ldots, I$;
(ii) $\pi_i(0) = 0$, $\forall i = 1, \ldots, I$;
(iii) $\pi_{i+1}(p) - \pi_i(p) \to \infty$ as $p \to \infty$, $\forall i = 1, \ldots, I - 1$.

For any p, let $m(p)$ be defined by the condition

$$V_{n+}^*(p, m(p)) = \sum_{i=1}^{I} a_{in+} V(\pi_i(p) + m(p)) = \bar{v}_{n+}.$$

It is clear from the properties of the utility function that $m(p)$ is well defined and continuous. Putting $p = 0$, we have

$$V_{n-}^*(0, m(0)) = V_{n+}^*(0, m(0)) = \bar{v}_{n+} > \bar{v}_{n-}.$$

From an earlier result

$$V_{n\cdot}^*(p, m(p)) = V(\pi_1(p) + m(p)) + \sum_{i=1}^{I-1} (V(\pi_{i+1}(p) + m(p)) - V(\pi_i(p) + m(p)))\,(1 - A_{in\cdot}).$$

From this it follows that

$$V_{n+}^*(p, m(p)) - V_{n-}^*(p, m(p)) = \sum_{i=1}^{I-1} (V(\pi_{i+1}(p) + m(p)) - V(\pi_i(p) + m(p)))\,(A_{in-} - A_{in+}).$$

By Assumption 5, $A_{in-} \geq A_{in+}$ for all i and the hypothesis $\bar{v}_{n+} > \bar{v}_{n-}$ implies that $A_{in-} - A_{in+} > 0$ for some i. Then as $p \to \infty$, the right-hand side diverges to ∞. Thus, for p sufficiently large, $V^*_{n+}(p, m(p)) - V^*_{n-}(p, m(p)) > \bar{v}_{n+} - \bar{v}_{n-}$, which implies that

$$V^*_{n-}(p, m(p)) = V^*_{n+}(p, m(p)) - \{V^*_{n+}(p, m(p)) - V^*_{n-}(p, m(p))\}$$
$$< \bar{v}_{n+} - (\bar{v}_{n+} - \bar{v}_{n-}) = \bar{v}_{n-}.$$

By continuity, there exists an intermediate value of p such that $V^*_{n-}(p, m(p)) = \bar{v}_{n-}$, as required.

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